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# On gravitational dressing of renormalization group $\beta$ -functions beyond lowest order of perturbation theory

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## Abstract

Based on considerations in conformal gauge I derive up to *nextleading* order a relation between the coefficients of  $\beta$ -functions in 2D renormalizable field theories before and after coupling to quantized gravity. The result implies a coupling constant dependence of the ratio of both  $\beta$ -functions beyond leading order.

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# 1 Introduction

The coupling of 2D conformal field theories to quantized 2D gravity (gravitational dressing) is well understood [1, 2, 3, 4], at least for central charges  $c \leq 1$  or  $c \geq 25$ . Only recently there has been some progress in the general discussion of gravitational dressing of the larger class of renormalizable 2D field theories. One considers a 2D theory described by the action

$$S = S_c + \sum_i g_i \int V_i d^2z , \quad (1)$$

where  $S_c$  is the action of a conformal field theory with central charge  $c$ ,  $V_i$  a set of marginal operators with respect to  $S_c$  which is closed under renormalization and  $g_i$  dimensionless couplings. Then it has been shown [5, 6, 7, 8] that the gravitational dressed RG  $\beta$ -functions  $\bar{\beta}_i(g)$  in *lowest order* are related to the original  $\beta$ -functions  $\beta_i(g)$  corresponding to the action  $S$  by the universal formula

$$\bar{\beta}_i(g) = \frac{2}{\alpha Q} \beta_i(g) . \quad (2)$$

$\alpha$  and  $Q$  are fixed by the central charge  $c$  of the unperturbed theory  $S_c$

$$Q = \sqrt{\frac{25-c}{3}} \quad (3)$$

$$\alpha = \frac{Q - \sqrt{Q^2 - 8}}{2} . \quad (4)$$

In its generalization to the case of an infinite number of couplings i.e. to generalized  $\sigma$ -models the dressing problem addresses the question: What critical (d+1)-dimensional string is the gravitational dressed version of what non-critical d-dimensional string?

The aim of this letter is the extension of the  $\beta \leftrightarrow \bar{\beta}$  relation to the next order of perturbation theory. Just the nextleading order is still of considerable interest since at least in mass-independent renormalization schemes for the simplest case of only one coupling the first two orders are scheme independent. In section 2 we will show that for the generic case eq.(2) cannot be valid if higher orders are included. This serves as a further motivation to look in section 3 for a formula connecting the nextleading orders of  $\beta$  and  $\bar{\beta}$ .

## 2 On the coupling constant dependence of the ratio $\beta$ to $\bar{\beta}$

For both  $\beta$  and  $\bar{\beta}$  we assume the perturbative structure <sup>2</sup>

$$\beta_i = \beta_i^{(2)ab} g_a g_b + \beta_i^{(3)abc} g_a g_b g_c + \dots$$

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<sup>2</sup>The extension to perturbations including relevant operators corresponding to linear terms in the  $\beta$ -functions is straightforward [5], but will not be discussed further.

$$\bar{\beta}_i = \bar{\beta}_i^{(2)ab} g_a g_b + \bar{\beta}_i^{(3)abc} g_a g_b g_c + \dots \quad (5)$$

Up to now it has been proven that

$$\bar{\beta}_i^{(n)} = \frac{2}{\alpha Q} \beta_i^{(n)} \quad \text{for } n = 2. \quad (6)$$

We want to argue that eq.(6) cannot be extended to all  $n$ . For this purpose we assume the existence of a theory for which the RG fixed point at  $g_i = 0$  is connected to a second RG fixed point at  $g_i = g_i^*$  via a RG trajectory ( $\beta_i \neq 0$  for  $0 < g_i < g_i^*$ ). Then

$$S_{c_\star} = S_c + \sum_i g_i^* \int V_i d^2 z \quad (7)$$

describes a conformal theory with central charge  $c_\star$ . Due to the c-theorem [10]  $c_\star$  differs from  $c$

$$c_\star \neq c. \quad (8)$$

If we further assume  $\left. \frac{\partial \beta_i}{\partial g_j} \right|_{g=g^*} = 0$  the expansion around  $g = g^*$  starts like that around  $g = 0$  with quadratic terms. We can treat the problem symmetrically as a perturbation around both points in coupling constant space. If (2) would be valid for the complete  $\beta$ -functions this would produce two different  $\bar{\beta}$ -functions:  $\frac{2}{\alpha Q} \beta$  and  $\frac{2}{\alpha_\star Q_\star} \beta$  ( $\alpha_\star$ ,  $Q_\star$  related to  $c_\star$  in analogy to (3), (4)).

A universal factor relating  $\beta$  to  $\bar{\beta}$  can be interpreted simply as a change in the renormalization scale  $\mu \rightarrow \bar{\mu} = \mu^\kappa$  [8]

$$\bar{\beta}(g) = \bar{\mu} \frac{\partial}{\partial \bar{\mu}} g = \frac{\bar{\mu}}{\mu} \frac{\partial \mu}{\partial \bar{\mu}} \beta(g) = \frac{1}{\kappa} \beta(g). \quad (9)$$

Along this line of argument the difference between the two  $\bar{\beta}$ -functions would be due to the use of different scales. One scale is defined by means of the fixed point theory at  $g = 0$ , the other one by the fixed point theory at  $g = g^*$ . Although this interpretation looks similar to a RG-scheme dependence, it is completely unsatisfactory.

At every point of the RG trajectory  $g \neq 0$ ,  $g \neq g^*$  the gravitational dressed scale has to be defined just by means of the theory under discussion. This requirement is of course fulfilled by the most natural choice of scale given by the use of a cutoff in the geodesic distance.

Insisting on a unique  $\bar{\beta}$ -function <sup>3</sup> the lowest order result (6) applied to the perturbation around both the  $c$  as well as the  $c_\star$  theory states for this  $\bar{\beta}$

$$\left. \frac{\partial^2 \bar{\beta}_i}{\partial g_k \partial g_l} \right|_{g=0} = \frac{2}{\alpha Q} \left. \frac{\partial^2 \beta_i}{\partial g_k \partial g_l} \right|_{g=0} \quad \text{and} \quad \left. \frac{\partial^2 \bar{\beta}_i}{\partial g_k \partial g_l} \right|_{g=g^*} = \frac{2}{\alpha_\star Q_\star} \left. \frac{\partial^2 \beta_i}{\partial g_k \partial g_l} \right|_{g=g^*}. \quad (10)$$

$\alpha Q \neq \alpha_\star Q_\star$  excludes an extension of (6) to all  $n$ , i.e. the ratio  $\frac{\bar{\beta}_i}{\beta_i}$  depends on  $g$ .

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<sup>3</sup>understood modulo standard scheme dependence

### 3 Gravitational dressing of the nextleading order contribution to $\beta$ -functions

We work in conformal gauge and use the method developed in refs. [5, 7]. It contains two steps. At first the gravitational dressed action is written as

$$\tilde{S} = S_c + S_L + \sum_i g_i \int \tilde{V}_i \sqrt{\hat{g}} d^2z . \quad (11)$$

Here

$$S_L[\phi|\hat{g}] = \frac{1}{8\pi} \int d^2z \sqrt{\hat{g}} (\hat{g}^{mn} \partial_m \phi \partial_n \phi + Q \hat{R} \phi(z) + m^2 e^{\alpha\phi}) \quad (12)$$

denotes the Liouville action for the Weyl degree of freedom of the two-dimensional metrics  $g_{ab} = e^{\alpha\phi} \hat{g}_{ab}$ .  $S_c + S_L$  just describes the gravitational dressed theory in the conformal case  $g_i = 0$  [2, 3]. For the dressing of the perturbations one makes the ansatz

$$\tilde{V}_i(z) = e^{\delta_i \phi(z)} V_i(z) . \quad (13)$$

Then the RG  $\beta$ -functions  $\tilde{\beta}_i$  for this theory, understood in the standard sense with respect to scaling in the coordinate  $z$ , are calculated as an expansion in  $g_i$  and  $\delta_i$ . The trace of the corresponding energy-momentum tensor is given by (including the contribution from diffeomorphism ghosts,  $(\tilde{V}_i)_r$  denoting the renormalized dressed perturbations)

$$\tilde{T}_a^a = \frac{c - 25 + 3Q^2}{24\pi} \hat{R} + \sum_i \tilde{\beta}_i (\tilde{V}_i)_r . \quad (14)$$

Strictly speaking this formula can be derived for  $m^2 = 0$  only. As is well known the Liouville interaction cannot be treated as a perturbation, i.e. it cannot be (with a corresponding  $\tilde{\beta}$ -function) included in the sum of the r.h.s. easily. In analogy to the handling of this problem in the case  $g_i = 0$  we assume that eq.(14) remains valid for  $m^2 \neq 0$  if  $\alpha$  is fixed by the requirement that  $e^{\alpha\phi}$  is a (1,1) operator. This may imply a dependence on  $g_i$ , see below.

The vanishing of  $\tilde{T}_a^a$ , i.e. independence of the scale of  $\hat{g}_{ab}$  requires (3) also for  $g \neq 0$  as well as  $\tilde{\beta}_i = 0$ . The last condition can be satisfied by treating the  $\delta_i$  as functions of the  $g_i$ .

After this construction of the gravitational dressed action  $\tilde{S}$ , in a second step, one defines the gravitational dressed RG function  $\tilde{\beta}_i$  in connection to the response of a change of the cutoff in the geodesic distance. The procedure makes use of the relation between coefficients in  $\beta$ -functions and operator product expansion (OPE) coefficients [9, 10, 11]. A direct extension to the next order has to include the influence of the dressing by Liouville exponentials on the OPE coefficients.

We found it convenient to use the arguments in a little bit implicit form by studying the perturbative expansion of two-point functions. A similar strategy has been used in refs.[12, 13]. For clarity we consider the one coupling case only and leave the straightforward

generalization to several couplings for further work.

Correlation functions of  $\tilde{V}$  with respect to the dressed unperturbed action  $S_c + S_L$  factorize in a conformal matter part and a Liouville part. The scaling dimension of  $V$  is two by assumption, that of  $e^{\delta\phi}$  is equal to  $-y$ , with  $y$  defined by

$$y = \delta(\delta - Q) . \quad (15)$$

This leads to

$$\langle \tilde{V}(z_1) \tilde{V}(z_2) \rangle_0 = \frac{A_2(\delta)}{|z_1 - z_2|^{4-2y}} , \quad (16)$$

$$\langle \tilde{V}(z_1) \tilde{V}(z_2) \tilde{V}(z_3) \rangle_0 = \frac{f \cdot A_3(\delta, \delta, \delta)}{(|z_1 - z_2| |z_1 - z_3| |z_2 - z_3|)^{2-y}} . \quad (17)$$

The constant  $f$  parametrizes the 3-point function of the undressed  $V$ , it appears also in the OPE  $V(z_1)V(z_2) = \frac{f}{|z_1 - z_2|^2} V(z_2) + \dots$

$A_2$  and  $A_3$  are the  $z$  independent factors in the 2 and 3-point functions of Liouville exponentials [14]. Using these building blocks the full 2-point function

$$\langle \tilde{V}(z_1) \tilde{V}(z_2) \rangle = \langle \tilde{V}(z_1) \tilde{V}(z_2) \rangle_0 - g \int d^2 z \langle \tilde{V}(z_1) \tilde{V}(z_2) \tilde{V}(z) \rangle_0 + O(g^2) \quad (18)$$

becomes

$$\langle \tilde{V}(z_1) \tilde{V}(z_2) \rangle = \frac{A_2(\delta)}{|z_1 - z_2|^{4-2y}} - \frac{\pi g f A_3(\delta, \delta, \delta)}{|z_1 - z_2|^{4-3y}} \frac{\Gamma(1-y)(\Gamma(\frac{y}{2}))^2}{\Gamma(y)(\Gamma(1-\frac{y}{2}))^2} + O(g^2) . \quad (19)$$

Obviously  $y > 0$  acts as a regulator. We now determine the renormalization  $Z$ -factors as functions of  $y$  and  $g$ . If  $\mu$  is the RG-scale, then the renormalized operator and coupling are defined by

$$\tilde{V} = Z_{\tilde{V}} \mu^{-y} \tilde{V}_r \quad (20)$$

$$g = Z_g \mu^y g_r , \quad (21)$$

respectively. From  $\int d^2 z \sqrt{\tilde{g}} \tilde{V}_r = \frac{\partial \tilde{S}}{\partial g_r}$  one finds the relation

$$Z_{\tilde{V}}^{-1} = Z_g + g_r \frac{\partial Z_g}{\partial g_r} . \quad (22)$$

Out of  $Z_g$  the wanted  $\beta$ -function  $\tilde{\beta}$  is determined by [17, 12]

$$\tilde{\beta} = \frac{-y g_r}{1 + g_r \frac{\partial \log Z_g}{\partial g_r}} . \quad (23)$$

Using (22) and

$$\frac{\Gamma(1-y)(\Gamma(\frac{y}{2}))^2}{\Gamma(y)(\Gamma(1-\frac{y}{2}))^2} = \frac{4}{y} (1 + O(y^2)) \quad (24)$$

the renormalized 2-point function can be written as

$$\begin{aligned} \langle \tilde{V}(z_1) \tilde{V}(z_2) \rangle_r &= \mu^{2y} \frac{A_2}{|z_1 - z_2|^{4-2y}} \left( Z_g + g_r \frac{\partial Z_g}{\partial g_r} \right)^2 \\ &\quad \left( 1 - (\mu|z_1 - z_2|)^y g_r Z_g \frac{4\pi f A_3}{y A_2} (1 + O(y^2)) + O(g_r^2) \right) . \end{aligned} \quad (25)$$

Now  $Z_g$  has to be chosen in a way to ensure finite r.h.s. for  $y \rightarrow 0$ .

The most delicate point in this construction is the behaviour of  $A_2$  and  $A_3$  for  $\delta \rightarrow 0$ . Using the explicit formulas of ref. [14]<sup>4</sup> one gets a regular behaviour of  $A_2(\delta)$  at  $\delta = 0$  but finds a divergence for  $A_3(\delta, \delta, \delta)$  at  $\delta = 0$ . This pattern is connected with Möbius invariance properties. To get a finite 2-point function for Liouville exponentials a division by the volume  $V_{CKV}^{(2)}$  of the subgroup of the Möbius group leaving  $z_1$  and  $z_2$  fixed is understood. This of course has to be done for all terms on the r.h.s. of (19), i.e. we must replace  $A_3$  by  $\frac{A_3}{V_{CKV}^{(2)}}$ . To give a meaning to this formal quantity,  $V_{CKV}^{(2)}$  should be regularized. However, since it is of no help to introduce a second independent regularization, the only way to treat the problem rigorously would consist in regularizing both  $V_{CKV}^{(2)}$  and the usual UV-divergencies [15, 16] by means of a regularized propagator for the Liouville field. Such a calculation has not been done. Nevertheless we can expect a finite answer for  $\frac{A_3}{V_{CKV}^{(2)}}$ . Altogether, taking into account a proper normalization of  $V_{CKV}^{(2)}$  we replace  $\frac{A_3}{A_2}$  in (25) by  $1 + O(\delta)$ .

Writing  $Z_g$  in the form

$$Z_g = 1 + b g_r + O(g_r^2) \quad (26)$$

finite r.h.s. of (25) up to  $O(g_r^2)$  is realized if

$$\left( 1 + 4b g_r + O(g_r^2) \right) \left( 1 - \frac{4\pi f}{y} (1 + O(y)) g_r + O(g_r^2) \right)$$

is finite up to  $O(g_r^2)$ . From this we conclude

$$b = \frac{\pi f}{y} \quad (27)$$

in a minimal subtraction scheme. In connection with (26) and (23) this yields

$$\tilde{\beta}(y, g_r) = -y g_r + \tilde{\beta}^{(2)}(y) g_r^2 + \tilde{\beta}^{(3)}(y) g_r^3 + O(g_r^4) \quad (28)$$

where

$$\tilde{\beta}^{(2)}(y) = \pi f = \beta^{(2)} . \quad (29)$$

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<sup>4</sup>There is a change in notation. We call here the Liouville mass  $m$  and the RG scale  $\mu$ .

In writing the second equation in (29) we have taken into account  $\tilde{\beta}(0, g_r) = \beta(g_r)$  which is due to the decoupling of the Liouville field for  $y = 0$ .

To handle the next leading gravitational dressing problem it is sufficient to know from (29) the  $y$ -independence of  $\tilde{\beta}^{(2)}$  and to assume regularity in  $y$  for  $\tilde{\beta}^{(3)}$  i.e. <sup>5</sup>

$$\tilde{\beta}^{(3)}(y) = \beta^{(3)} + O(y) . \quad (30)$$

Then the requirement  $\tilde{\beta} = 0$  determines

$$y = \beta^{(2)} g_r + \beta^{(3)} g_r^2 + O(g_r^3) , \quad (31)$$

which via (15) implies for  $\delta$

$$\delta = -\frac{\beta^{(2)}}{Q} g_r + \frac{1}{Q} \left( \frac{(\beta^{(2)})^2}{Q^2} - \beta^{(3)} \right) g_r^2 + O(g_r^3) . \quad (32)$$

Let us turn to the cutoff regularized version of  $\tilde{S}$  and denote the cutoff in geodesic distance by  $a$ . It appears in the combination  $a^2 \cdot e^{-\alpha\phi}$  only. This expression is proportional to the squared cutoff  $l^2$  in coordinate space, i.e. <sup>6</sup>

$$a^2 = \hat{g}_n^n l^2 e^{\alpha\phi} . \quad (33)$$

A variation of  $a$  can be compensated by a shift of the *constant* mode of the Liouville field related by

$$2 \frac{da}{a} = \alpha d\phi . \quad (34)$$

This shift influences the linear term in  $S_L$ , the Liouville interaction term  $m^2 e^{\alpha\phi}$  and the dressed interaction term  $g\tilde{V}$ . The linear term, due to the Gauß-Bonnet theorem, causes an overall factor in front of the functional integral which drops out if normalization by the partition function is taken into account. The Liouville interaction term remains unchanged if the shift of the Liouville field is compensated by a change of the Liouville mass  $m$

$$2 \frac{dm}{m} = -\alpha d\phi . \quad (35)$$

Finally, requiring invariance also for the dressed interaction term yields information on the dependence of the bare coupling  $g$  on  $m$  and  $a$  at fixed  $g_r$

$$0 = d(g e^{\delta\phi} V) \Big|_{g_r} = \left( \frac{\partial g}{\partial a} da + \frac{\partial g}{\partial m} dm + g \delta d\phi \right) e^{\delta\phi} V .$$

With (34),(35) this gives

$$\frac{m \partial g}{\partial m} - \frac{a \partial g}{\partial a} = \frac{2g\delta}{\alpha} . \quad (36)$$

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<sup>5</sup>Similar to dimensional regularization one could expect independence of  $y$  for all  $\tilde{\beta}^{(n)}$ ,  $n \geq 2$ . However, for us it is sufficient to assume the weaker property (30).

<sup>6</sup>In this relation  $\phi$  has to be understood as the Liouville integration variable in the functional integral.

The solution of this equation is

$$g = \left( \frac{m}{\bar{\mu}} \right)^{\frac{\delta}{\alpha}} (a\bar{\mu})^{-\frac{\delta}{\alpha}} f(am) g_r \quad , \quad (37)$$

$\bar{\mu}$  is a RG-scale, necessary for dimensional reasons.

The limit  $a \rightarrow 0$  exists by construction since  $y > 0$  acts as an effective regularization parameter.<sup>7</sup> This fixes  $f(am) = (am)^{\frac{\delta}{\alpha}}$ , i.e.

$$g = \left( \frac{m}{\bar{\mu}} \right)^{\frac{2\delta(g_r)}{\alpha}} g_r \quad . \quad (38)$$

The gravitational dressed  $\beta$ -function is now defined by

$$\bar{\beta} = m \left. \frac{\partial g_r}{\partial m} \right|_{\bar{\mu}, g \text{ fix}} \quad . \quad (39)$$

With (32) this yields

$$\begin{aligned} \bar{\beta}(g_r, \frac{m}{\bar{\mu}}) &= \frac{2\beta^{(2)}}{\alpha Q} g_r^2 + \frac{2}{\alpha Q} \left( \beta^{(3)} - \frac{(\beta^{(2)})^2}{Q^2} \right) g_r^3 \\ &+ \frac{4(\beta^{(2)})^2}{\alpha^2 Q^2} g_r^3 \log \frac{m}{\bar{\mu}} + O(g_r^4) \quad . \end{aligned} \quad (40)$$

We still have to consider a possible  $g_r$  dependence of  $\alpha$ . As discussed in connection with the trace formula (14)  $\alpha$  should be fixed by requiring scaling dimension 2 for  $e^{\alpha\phi}$  also in the case  $g \neq 0$ . If one studies the two-point function of this operator in analogy to that of  $\tilde{V}$ , due to  $\langle \tilde{V} \rangle_0 = 0$ , the perturbative corrections start with  $O(g^2)$ . Therefore, a  $g_r$ -dependence of  $\alpha$  can influence  $\bar{\beta}$  in order  $O(g_r^4)$  only.

The dependence of  $\bar{\beta}$  on  $\frac{m}{\bar{\mu}}$  fits into the standard situation for  $\beta$ -functions in massive theories. By a mass dependent redefinition of  $g_r$  one can switch to a RG scheme with a mass independent  $\bar{\beta}$ -function. In the class of mass independent schemes  $\bar{\beta}^{(3)}$  is unambiguously given by

$$\bar{\beta}^{(3)} = \frac{2}{\alpha Q} \left( \beta^{(3)} - \frac{(\beta^{(2)})^2}{Q^2} \right) \quad . \quad (41)$$

A last comment concerns the overall sign of  $\bar{\beta}$ . Replacing  $m \frac{\partial}{\partial m}$  in (39) by the more conventionally looking  $\bar{\mu} \frac{\partial}{\partial \bar{\mu}}$  would result in an overall minus sign. However, since the overall sign is fixed by the quasiclassical limit ( $Q \rightarrow \infty$ ,  $\alpha Q \rightarrow 2$ ) this sign cannot be accepted. One should mention that also the dressed dimensions in the KPZ relation are equal to the power of  $m$  and the negative power of  $\bar{\mu}$  ( see [2, 3] and second ref. of [4]).

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<sup>7</sup> $y$  is fixed by (31) already. This seems to restrict the method to the subset of theories yielding positive  $y$ . However, without any doubt the final result is valid for both signs of  $y$ .

## 4 Concluding remarks

Looking at our result (41) from a more thorough point of view the formal treatment of the Möbius divergency appears as a weak spot in its derivation. Further work should improve the status of the argument. In parallel the result can be tested by calculations in concrete models. One candidate for the simple one-coupling case is the Gross-Neveu model coupled to gravity. The undressed  $\beta$ -function is known up to  $O(g^4)$  [18]. Other tests or applications are connected with the study of continuum limits of 2D lattice models on random lattices versus standard lattices. In addition one should search for general relations between dressed and undressed  $\beta$ -functions for generalized  $\sigma$ -models.

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